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LETTER TO THE EDITOR

Accelerating 't Hooft–Polyakov monopole

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Abstract. The fields near a slowly accelerating 't Hooft–Polyakov monopole in the Prasad–Sommerfield limit are obtained.

We consider the motion of the slowly accelerating 't Hooft–Polyakov monopole in the Prasad–Sommerfield limit. Following Manton (1977), we assume that the time dependence of the Higgs field φ^a and the gauge field A^a_μ which describe the monopole is

$$\begin{aligned} \varphi^a(\boldsymbol{x}, t) &= \varphi^a(\boldsymbol{x} - \tfrac{1}{2}\boldsymbol{w}t^2), \\ A^a_i(\boldsymbol{x}, t) &= A^a_i(\boldsymbol{x} - \tfrac{1}{2}\boldsymbol{w}t^2), \end{aligned} \tag{1}$$

where the acceleration w is a small constant. For convenience, the direction of $\boldsymbol{w}$ will be taken along the x_1 axis. All the terms of order w^2 or higher will be neglected. In an instantaneous rest frame, we assume that $A^a_0 = 0$, so in a non-accelerating frame $A^a_0 = -wtA^a_1$. It can be proved that if φ^a and A^a_i satisfy the equation

$$F^a_{ij} = -\varepsilon^{ijk}(D_k + \delta_{k1}w)\varphi^a, \tag{2}$$

then the field equations will be satisfied, i.e., equation (2) is the sufficient condition of φ^a and A^a_μ to satisfy

$$\begin{aligned} D^\mu F^a_{\mu\nu} &= -\varepsilon^{abc}D_\nu\varphi^b\varphi^c, \\ D^\mu D_\mu\varphi^a &= 0. \end{aligned} \tag{3}$$

In (2) and (3), $D_\mu\varphi^a = (\delta_{ab}\partial_\mu + \varepsilon^{abc}A^b_\mu)\varphi^c$.

We take an ansatz

$$\begin{aligned} \varphi^a &= r_aL + \delta_{a1}rN + r_ar_1M/r, \\ A^a_i &= \varepsilon^{aij}r_jK + \varepsilon^{aij}r_jr_1R/r + \varepsilon^{aj1}r_jr_iQ/r, \end{aligned} \tag{4}$$

where $r_i = x_i - \frac{1}{2}\delta_{i1}wt^2$. L and K are given by the static Prasad–Sommerfield solution (1975), i.e., $L = (cr\coth cr - 1)/r$ and $K = (1 - cr/\sinh cr)/r^2$, where c is a constant. M, N, R and Q are unknown functions of r, and will be determined by (2). They are all of order w. The φ^a and A^a_i in (4) represent a perturbative 't Hooft–Polyakov

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monopole. Substituting (4) in (2), we find:

$$\begin{aligned}
&(\mathrm{d}/\mathrm{d}r)(R/r)+(3R-Q-M)/r^2+K(M+N)+LR=0,\\
&(\mathrm{d}/\mathrm{d}r)Nr+r^2LQ+R=0,\\
&(\mathrm{d}/\mathrm{d}r)(M/r)+(\mathrm{d}/\mathrm{d}r)(R/r)+L(R-Q)+K(M+2R-Q)=0,\\
&M-Q+r^2K(N+Q)+rwL=0.
\end{aligned} \tag{5}$$

We have four equations to determine the four unknown functions M, N, Q and R which should be regular at $r=0$. We are unable to solve (5) exactly yet, but we can solve them approximately to the desirable accuracy near $r=0$. For small r ($cr<\pi$) L and K can be expanded in power series. We write $M=M_0+M_1r+M_2r^2+\ldots$, $N=N_0+N_1r+N_2r^2+\ldots$, $Q=Q_0+Q_1r+Q_2r^2+\ldots$, and $R=R_0+R_1r+R_2r^2+\ldots$, where M_i, N_i, Q_i and R_i are unknown constants. From (5) we have a set of linear equations to decide M_i, N_i, R_i and Q_i successively. From these linear equations we finally have

$$\begin{aligned}
\varphi^a &= r_aL+\tfrac{1}{6}wc^2\delta_{a1}r^2(1+\tfrac{1}{10}c^2r^2+\tfrac{1}{280}c^4r^4+\tfrac{1}{15120}c^6r^6+\ldots)\\
&\quad -\tfrac{2}{3}wc^2r_ar_1(1-\tfrac{7}{120}c^2r^2+\tfrac{121}{10080}c^4r^4-\tfrac{61}{43200}c^6r^6+\ldots),\\
A_i^a &= \varepsilon^{aij}r_jK-\tfrac{1}{3}wc^2\varepsilon^{aij}r_jr_1(1-\tfrac{2}{15}c^2r^2+\tfrac{11}{504}c^4r^4-\tfrac{107}{37800}c^6r^6+\ldots)\\
&\quad -\tfrac{1}{3}wc^2\varepsilon^{aj1}r_ir_j(1+\tfrac{1}{30}c^2r^2+\tfrac{13}{2520}c^4r^4-\tfrac{1}{3600}c^6r^6+\ldots).
\end{aligned} \tag{6}$$

The corresponding magnetic field is

$$\boldsymbol{B}=-\left(1-\frac{\boldsymbol{w}\cdot\boldsymbol{r}}{2}\right)\frac{\boldsymbol{r}}{r^3}+\frac{\boldsymbol{w}}{2r}. \tag{7}$$

φ^a and A_i^a are regular at $r=0$, and the total magnetic flux is -4π. We note that the result (7) is not dependent on how many terms taken from the power series in (6) are used to calculate $\boldsymbol{B}$. Both more and less terms give the same expression for $\boldsymbol{B}$.

A monopole can be accelerated by a magnetic field; it can also be accelerated by a Higgs field. That the Higgs field can offer a long-range force on a monopole is obvious from the existence of the static N-monopole solutions (Ward 1981, Corrigan and Goddard 1981). How about our solution? If the monopole is accelerated by a magnetic field, this magnetic field should be included in (7). However, the $\boldsymbol{B}$ in (7) is just the field generated by a slowly accelerating magnetic monopole according to Maxwell's equations. It is clear that the 't Hooft–Polyakov monopole which is discussed by us is not accelerated by a magnetic field. The solution seems to describe a monopole accelerated by the Higgs field.

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